

## General Locked Rotation Function

### Program GLRF

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## INTRODUCTION

Given two homologous molecules, A and B, the rotation ( $[\rho]$ ) that brings them into the same orientation will also bring the inter-atomic vectors of each molecule into overlap. Crystallographically, the set of inter-atomic vectors is represented by the Patterson function. Hence, an ordinary rotation function (M. G. Rossmann & D. M. Blow. *Acta Cryst.* **15**, 24–31, (1962)) can be defined as the overlap of one Patterson function and the rotated version of another Patterson function,

$$RF([C]) = \int_{\Omega} P_A(u)P_B([C]u)du \quad (1)$$

where

$$[C] = [\alpha_B][\rho][\beta_A] \quad (2)$$

and

$$x_B = [C]x_A + d \quad (3)$$

$[\alpha]$  is the deorthogonalization matrix which converts Ångstrom coordinates relative to a Cartesian system to fractional coordinates relative to the crystal unit cell.  $[\beta]$ , the inverse of  $[\alpha]$ , is the orthogonalization matrix. The integration volume ( $\Omega$ ) is usually a sphere centered at the Patterson origin. The radius ( $R$ ) of this sphere can be chosen to exclude most of the vectors between atoms in different molecules in the crystal, which are generally longer than the inter-atomic vectors within one molecule.

The rotation  $[\rho]$  is usually represented by a set of polar ( $\phi, \psi, \kappa$ ) or Eulerian ( $\theta_1, \theta_2, \theta_3$ ) angles. In the case of polar angles,  $\phi$  and  $\psi$  define the orientation of a rotation axis relative to a Cartesian coordinate system and  $\kappa$  defines the angle of rotation around this axis. In the case of Eulerian angles, the three angles define a sequential set of rotations around the  $X, Y, Z$  axes of a Cartesian coordinate system. For both polar and Eulerian angles, the actual meaning of each angle depends on the convention that is used.

The function  $RF$  is generally expected to reach its maximum when the rotation  $[\rho]$  corresponds to the rotation that brings the two molecules to the same orientation. In practice, a grid search is carried out covering the entire unique region of the rotational space and Eq. (1) is evaluated at each grid point. The grid point that gives the highest value for Eq. (1) is then possibly the correct rotation. The rotation function values at other grid points that are evaluated in such a search give an estimate for the background noise in the calculation.

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Reference: L. Tong & M. G. Rossmann *Acta Cryst.* **A46**, 783–792, (1990).

Reference: L. Tong & M. G. Rossmann *Methods in Enzymology* **276**, 594–611, (1997).

Rotation functions can be classified by the way Eq. (1) is evaluated (for example, the slow and the fast rotation functions) or by whether the two molecules reside in the same crystal (self and cross rotation functions).

## I. THE SLOW ROTATION FUNCTION

Eq. (1) can be evaluated in many ways in practice. If both Patterson functions are expressed as their individual Fourier transforms,

$$P_A(u) = \sum_h F_h^2 e^{-2\pi i h u}$$

$$P_B(u) = \sum_p F_p^2 e^{-2\pi i p u}$$

it can be shown (M. G. Rossmann & D. M. Blow. *Acta Cryst.* **15**, 24–31, (1962)) that

$$RF([C]) = \sum_h \sum_p F_h^2 F_p^2 G_{hp} \quad (4)$$

where

$$G_{hp} = \int_{\Omega} e^{-2\pi i (h+p[C])u} du \quad (5)$$

$$= \frac{3(\sin 2\pi H R - 2\pi H R \cos 2\pi H R)}{(2\pi H R)^3} \quad (6)$$

is the  $G$  function, which represents the Fourier transform of a sphere of radius  $R$ .  $H$  is the length of the reciprocal space vector  $h + p[C]$ . The function assumes the maximum value of 1 when  $h + p[C] = 0$  and it quickly approaches and then oscillates around zero as  $h + p[C]$  deviates from zero.

Eq. (4) is sometimes referred to as the ‘slow’ rotation function, as its evaluation is generally time-consuming. Several techniques have been developed to speed up the calculations of the slow rotation function. The rotation function value is proportional to the intensity of the reflections (Eq. (4)). It can then be expected that rotation function values are dominated by strong reflections. Therefore, only the strong reflections of crystal B ( $p$ ) are used in the calculation. These reflections, also known as the large terms (P. Tollin & M. G. Rossmann. *Acta Cryst.* **21**, 872–876, (1966)), are selected based on the criterion

$$I_p \geq C \times \langle I_p \rangle$$

where  $C$  is the cut-off value and the average intensity  $\langle I_p \rangle$  is generally calculated in shells of equal reciprocal volume. A cut-off value ( $C$ ) of 1.5 usually selects about 20% of the reflections as large terms.

Given the large terms of crystal B, the summation over the reflections of crystal A ( $h$ ) can be limited to those which make  $h + p[C]$  small, as otherwise the value of the  $G$  function will be negligible. For each large term ( $p$ ) in crystal B, only reflections in crystal A that are within an interpolation box centered on the reciprocal lattice point closest to  $-p[C]$  are used in the summation. The size of this interpolation box is generally  $3$  (reciprocal lattice points)  $\times 3 \times 3$ .

The calculation of the  $G$  function value based on Eq. (6) at each rotation search grid point is tedious. Tables of the  $G$  function values can be set up beforehand, either in reference to the value of  $H$  or in reference to the  $h, k, l$  index in the interpolation box, to speed up the calculation.

The values of the rotation function vary relatively slowly as a function of the rotation angles. If a reasonably fine search grid is used, it will be unlikely for the neighbors of a grid point to have high rotation function values if the grid point itself has a low rotation function value. Therefore, the evaluation of rotation functions using Eq. (4) can be accelerated by ignoring such grid points in the calculations (L. Tong. *J. Appl. Cryst.* **26**, 748–751, (1993)).

## II. THE FAST ROTATION FUNCTION

Even with the improvements described, the calculation of the slow rotation function is still rather time-consuming. The fast rotation function (R. A. Crowther. *The Molecular Replacement Method*, pp 173–178, Ed. M. G. Rossmann. Gordon & Breach, New York) uses a different approach to evaluate Eq. (1). The Patterson function is expanded in terms of the spherical harmonics,

$$S_{lmn}(r, \theta, \phi) = j_l(k_{ln}r)Y_l^m(\theta, \phi)$$

and

$$P_A(r, \theta, \phi) = \sum_{lmn} a_{lmn} S_{lmn} \quad (7)$$

$$P_B(r, \theta, \phi) = \sum_{l'm'n'} b_{l'm'n'}^* S_{l'm'n'}^* \quad (8)$$

where  $j_l$  is the normalized spherical Bessel function of order  $l$  and  $k_{ln}$  is chosen such that  $j_l(k_{ln}R) = 0$ .  $Y_l^m$  is the normalized spherical harmonics function. The expansion coefficient  $a_{lmn}$  can be calculated from

$$a_{lmn} = \sum_h F_h^2 T_{lmn}(h) \quad (9)$$

where  $T_{lmn}$  is the Fourier transform of  $S_{lmn}$ .

With such an expansion for both Patterson functions and utilizing the special properties of the spherical harmonics, Eq. (1) can be simplified to

$$RF(\theta_1, \theta_2, \theta_3) = \sum_m \sum_{m'} f_{mm'}(\theta_2) e^{-im\theta_1} e^{-im'\theta_3} \quad (10)$$

where the rotation is represented by a set of Eulerian angles  $(\theta_1, \theta_2, \theta_3)$ , and the function  $f_{mm'}$  is only dependent on  $\theta_2$ . Eq. (10) is a two-dimensional Fourier transform over  $\theta_1$  and  $\theta_3$ . Therefore, for any given  $\theta_2$ , the equation can be evaluated by the fast Fourier transform (FFT) technique, thus greatly accelerating the rotation function calculation.

The calculation of the expansion coefficients  $a_{lmn}$ , however, does still require some amount of computing time. The large term approach as developed for the slow rotation function can also be employed here. Test calculations have shown that a much smaller cut-off ( $C$ ) value should be used (roughly 0.1 to 0.5) to obtain better rotation function results.

## III. THE ORDINARY SELF ROTATION FUNCTION

When the two molecules A and B exist in the same crystal, a self rotation function can be used to determine the orientation and the angle of rotation of the local symmetry axis relating the two molecules.

Polar angles are usually preferred for representing the rotation  $[\rho]$  in ordinary self rotation functions. For example, if the molecules are related by a two-fold axis,  $\kappa$  can be fixed at  $180^\circ$  and the rotation search can be limited to the  $\phi$  and  $\psi$  angles. The search results can then be presented as a stereographic projection.

If the two molecules are related by an improper rotation (*i.e.*, an angle of rotation not divisible to 360), a full three-dimensional rotation search may be necessary. It is usually more advantageous to represent the rotation  $[\rho]$  as a set of Eulerian angles in such a case. The unique region of rotational space for different non-cubic space group symmetries is given by S. N. Rao, J.-H. Jih & J. A. Hartsuck. *Acta Cryst.* **A36**, 878–884, (1980). Once the rotation  $[\rho]$  is known, a set of polar angles can be extracted from it.

The ordinary rotation function will always have a maximum value at the origin ( $\kappa = 0$  for polar angles,  $\theta_2 = 0, \theta_1 + \theta_3 = 0$  for Eulerian angles) corresponding to the overlap of the Patterson function onto itself. This value can be set to be a specific constant (for example, 1000) and a scale factor is then applied to all the rotation function values to make the maximum 1000. This will bring the self rotation function to an ‘absolute’ scale. The rotations corresponding to the crystallographic symmetry elements should all produce rotation function values of 1000.

#### IV. THE ORDINARY CROSS ROTATION FUNCTION

If the two molecules are in two separate crystals, the ordinary cross rotation function can be used to determine the relative orientation of the two molecules. The two crystals may have been prepared experimentally, or an artificial crystal can be prepared by placing a model, usually the structure of a homologous protein, into an arbitrary unit cell. In the latter case, the dimensions of the unit cell are usually chosen large enough so that the inter-molecular vectors are longer than the intra-molecular ones to avoid their interference in the rotation function calculations. Cell dimensions that are twice the dimensions of the model should generally suffice for this purpose.

Eulerian angles are generally preferred for the cross rotation function. The unique region of rotational space for non-cubic space group symmetries of the A and B crystals is given by S. N. Rao, J.-H. Jih & J. A. Hartsuck.

Sometimes it is known that a direction in crystal A is aligned with a direction in crystal B (for example, the alignment of the local two-fold axis of a dimer in two different crystal forms). In such a case, a pre-rotation can be used first to align the two directions. Subsequent rotation searches is then carried out in polar angles, varying only the angle  $\kappa$ .

#### V. THE LOCKED SELF ROTATION FUNCTION

Many macromolecular crystals contain assemblies of the macromolecules obeying a simple (local) point group symmetry. The ordinary self rotation function can be used to determine the orientation of each local symmetry axis individually. However, if the symmetry of the point group is known, orientations of all the symmetry elements of the point group can be determined at the same time, using the so-called locked self rotation function (L. Tong & M. G. Rossmann. *Acta Cryst.* **A46**, 783–792, (1990)). It assumes a point group symmetry for the macromolecular assembly and maintains that symmetry throughout its searches (*i.e.*, the local symmetry is *locked* to the one specified at the outset).

A standard orientation is defined for the local symmetry point group. For example, a 222 point group symmetry can be defined with its three two-fold axes along the  $X$ ,  $Y$  and  $Z$  axes of a Cartesian coordinate system. If  $[I_n](n = 1, \dots, N)$  represents the set of local symmetry rotation matrices in the standard orientation, the rotation matrices after a rotation  $[E]$  has been applied is given by

$$[\rho_n] = [E][I_n][E]^{-1} \quad (11)$$

An ordinary self rotation function value ( $R_n$ ) can be calculated corresponding to each of the rotation matrices. The locked rotation function value ( $R_L$ ) is defined as the average of the individual values,

$$R_L = \frac{1}{N-1} \sum_{n=2}^N R_n \quad (12)$$

The locked self rotation function calculations are generally carried out in terms of Eulerian angles. In special cases where a local symmetry axis is known to be aligned with a direction in the crystal, polar angles can be used.

The locked rotation function simplifies the self rotation searches for large macromolecular assemblies (for example, viruses). Moreover, it also leads to a reduction, by a factor of about  $\sqrt{N}$ , in the background noise level of the rotation function as compared to the ordinary self rotation function.

The slow rotation function can be employed to calculate the locked self rotation function. Alternatively, the fast rotation function can be used to calculate an ordinary rotation function covering the entire Eulerian space. The locked rotation function can then be calculated by interpolating among the computed ordinary rotation function values.

The locked self rotation function can also be applied to cases where the macromolecular assembly does not obey a simple point group. In such cases, however, it is usually difficult to define a standard orientation.

## VI. THE LOCKED CROSS ROTATION FUNCTION

The presence of local symmetry can also be utilized in the cross rotation searches if only the monomer is used as the model. For every rotation  $[E]$  that brings the search model into the same orientation as that of one of the molecules in the assembly, there should be a set of other rotations that will bring the search model into overlap with other molecules in the assembly. Assuming that  $[J_n](n = 1, \dots, N)$  are the local symmetry rotation matrices in crystal B, which can be determined from either ordinary or locked self rotation function calculations, the set of rotations is given by

$$[C_n] = [\alpha][J_n][E][\beta] \quad (13)$$

Therefore, a locked cross rotation function can be defined as the average of the individual ordinary cross rotation function values evaluated with the rotation matrices  $[C_n]$ ,

$$R_L = \frac{1}{N} \sum_n R_n \quad (14)$$

Alternatively, assume that  $[I_n](n = 1, \dots, N)$  are the local symmetry rotation matrices in a standard orientation,  $[F]$  is the rotation that brings the standard orientation to that of the assembly in the crystal, and

$[E]$  is the rotation that brings the monomer to the same orientation as one of the monomers in the standard orientation,

$$[\rho_n] = [F][I_n][E] \quad (15)$$

With this definition, it is easier to define the unique region of rotational space for a locked cross rotation function.

## VII. GENERAL STRATEGIES FOR ROTATION FUNCTION CALCULATIONS

Rotation function calculations usually proceed in two steps. In the first step, a search is carried out covering the unique region of the rotational space to find the sets of angles that maximize the rotation function. Self rotation function calculations are usually limited to sections of constant  $\kappa$  and the slow rotation function can be used, with grid intervals along  $\phi$  and  $\psi$  of 2 to 3°. For ordinary cross rotation function calculations, the fast rotation function should be used in this initial search. If the results from the fast rotation function is unsatisfactory, the slow rotation function should be tried (but it will take some more CPU time), as it uses a completely different approach. A relatively coarse search grid (generally 3 or 4° intervals) is needed to reduce the total number of grid points in the search.

Once an estimate for the set of angles is obtained, the rotation search should be repeated using finer grid intervals (0.5 to 1°). The slow rotation function is used for this purpose. The search should be limited to a small region (3 to 5°) around the values for the rotation angles as determined by the initial search. This will give the optimized values for the rotation angles.

Alternatively, a Patterson-correlation refinement can be used to optimize the angular parameters.

## VIII. SPECIALIZED TRANSLATION FUNCTIONS

If the local symmetry is a two-fold, the translation element along this axis  $d_{//}$  can be determined (M. G. Rossmann, D. M. Blow, M. M. Harding, E. Collier. *Acta Cryst.* **17**, 338–342, (1964)),

$$T_1(\Delta) = \sum_h \sum_p F_h^2 F_p^2 G_{hp} \cos 2\pi(h-p)\Delta \quad (16)$$

This calculation does not need the phase information of the reflections.

Once phase information is available, the location of the local symmetry axis in the unit cell can be determined based on the maximal electron-density overlap of the subunits related by the local symmetry (D. M. Blow, M. G. Rossmann, B. A. Jeffrey. *J. Mol. Biol.* **8**, 65–78, (1964); L. Tong. *J. Appl. Cryst.* **26**, 748–751, (1993)).

If  $s_A$  and  $s_B$  are the centers of the two molecules,

$$s_B = [C]s_A + d \quad (17)$$

The electron density overlap of the two molecules,

$$\int_{\Omega} \rho(x_A)\rho(x_B)dx_A$$

where the integration volume is a sphere of radius  $R$  centered at  $s_A$ , can be written as

$$T([C], s_A, s_B) = \sum_h \sum_p F_h e^{i\phi_h} F_p e^{i\phi_p} G_{hp} e^{-2\pi i h s_A} e^{-2\pi i p s_B} \quad (18)$$

Given the rotational relationship between molecules A and B and an estimate for the center of molecule A, the center of molecule B can be determined by a Fourier transform (Eq. 18). In this application of Eq. (18), the two molecules can reside either in the same crystal or in two different crystals.

Once initial estimates for  $[C]$ ,  $s_A$ ,  $s_B$  are available, Eq. (18) can be used to optimize these parameters. A fine search can be carried out around the current values for  $[C]$  and  $s_A$  (or  $s_B$ , only one of which needs to be varied in the search) to obtain values that will maximize the correlation.

If molecules A and B reside in the same crystal, the position of this local symmetry axis can be determined. Assuming  $s$  lies on this axis,

$$s = [C]s + d_{//} \quad (19)$$

where  $d_{//}$  is the translation element along this local symmetry axis (see Eq. 16). Eq. 18 then becomes

$$T_2(s) = \sum_h \sum_p F_h e^{i\phi_h} F_p e^{i\phi_p} G_{hp} e^{-2\pi i p d_{//}} e^{-2\pi i (h+p)s} \quad (20)$$

Therefore, the center of the local symmetry axis can be determined by a Fourier transform. This function will produce a long, sausage-shaped density corresponding to the position of the local symmetry axis. To obtain more accurate parameters for the position of the local symmetry element, Eq. (18) should be used. After the maximization of the overlap, the rotational parameters of the local symmetry axis is given by  $[C]$ . The position of the local symmetry axis is given by  $(s_A + s_B)/2$ , and  $d_{//}$  is given by the projection of  $s_B - s_A$  along the direction of the local symmetry axis.

## DESCRIPTION OF INPUT COMMANDS

Appendix A contains a listing of the new features that are available for the current version of the program. Appendix B contains example inputs.

All input commands to the program are keyword-based and free-formatted. The input parser converts lower case characters to upper case so the program commands are not case dependent (though the program keeps the cases of the file names and the title unchanged). Each input line can contain at most 80 characters. The following characters are recognized as delimiters between words – a space, a comma, a tab, and an equal sign. More delimiters can be implemented by inserting them in the array SPACER in subroutine PARSER (and change the variable NSP at the same time). Comments in the input can be introduced by using the COMMENT command or using the special character “!” — in the input parser, all characters in the input line beginning at the “!” are ignored.

Presently, the program uses five logical I/O units, labelled by the variables LIN, LOG, LPRT, LOBS, and LSCRCH, which are initialized to be 5, 6, 3, 1, and 2, respectively. (The default values of most of the input variables are initialized in subroutine INITSS). All input commands are read from unit LIN. The output of the program will be written to unit LPRT. A name can be specified for this print file by using the PRINT command. The reflection data (if any) are read from unit LOBS. This unit is also used internally to read and/or write rotation function map files. The scratch file on unit LSCRCH is used for echoing the input



**EULER**-Angle

EULER\*3

[ZYZ]

This specifies the definition convention of Eulerian angles. Two conventions are currently supported — ZXZ and ZYZ, corresponding to rotation around the Cartesian Z axis ( $\theta_3$ ), then around the X (and Y, respectively) axis ( $\theta_2$ ), and finally around the Z axis ( $\theta_1$ ). ZXZ is the convention described in M. G. Rossmann & D. M. Blow *Acta. Cryst.* **15**, 24, (1962). ZYZ is used in program MERLOT (P. M. D. Fitzgerald, *J. Appl. Cryst.* **21**, 273, (1988)). The angles should be input to the program in the following order —  $\theta_1, \theta_2, \theta_3$ , and the program outputs the angles in the same order.

**WARNING:** The matrix used in this program is the transpose of the matrix as printed in Table 1 of the paper by Rossmann & Blow. Therefore, if you are comparing the results from this program with other programs, be sure that the programs are using the matrices the same way! The same goes for the command POLAR (see below).

Consequently, the search limits for angles  $\theta_1$  and  $\theta_3$  as defined by S. N. Rao, J.H. Jih & J. A. Hartsuck (*Acta Cryst.* **A36**, 878–884, (1980)) should be swapped. The  $\theta_1$  limit should be used as the limit on  $\theta_3$  in this program. In any case, the user is strongly encouraged to check out the symmetry of the rotation function with the program. A few (1 or 2) large terms and very large intervals (5-10°) should be used to speed up the calculation.

**POLAR**-Angle

POLAR\*3

[XZK]

This specifies the polar angle definition convention. Two conventions are currently supported — XYK and XZK. In both cases,  $\phi$  is the angle from the Cartesian X axis. In convention XYK (as defined in Rossmann & Blow),  $\psi$  is the angle from the Y axis. In convention XZK (as in MERLOT),  $\psi$  is the angle from the Z axis.  $\kappa$  is the rotation around the axis defined by  $\phi$  and  $\psi$ . The angles should be input to the program in the following order —  $\phi, \psi, \kappa$ , and the program outputs the angles in the same order.

The XZK convention should be used for most calculations. The XYK convention can be used for monoclinic space groups in the  $b$ -unique setting.

**ORTH**ogonalization

ORDOR\*5

[AXABZ]

This specifies the orthogonalization convention that should be used by the program. Four conventions are currently supported — BYBCX, CZBCX, AXABZ, CXCAZ, and Y3XYC. In convention BYBCX, the real space  $b$  axis coincides with the Cartesian Y axis, and  $b \times c$  (or  $a^*$ ) coincides with the X axis (this is first defined in the paper by Rossmann & Blow). In convention CZBCX, the real space  $c$  axis coincides with the Cartesian Z axis, and  $b \times c$  (or  $a^*$ ) coincides with the X axis (this is IPER=1 in MERLOT). In convention AXABZ, the real space  $a$  axis coincides with the Cartesian X axis, and  $a \times b$  (or  $c^*$ ) coincides with the Z axis (this is used by the Protein Data Bank, FRODO's helper SAM, PROTEIN, and X-Plor. This is also option NCODE=1 in program ALMN). Convention CXCAZ corresponds to NCODE=3 in ALMN and is suggested by David Borhani. This convention should be used only for monoclinic space groups in the  $b$ -unique setting. Convention Y3XYC, courtesy of Dr. Murthy, has been designed for rhombohedral space groups in the rhombohedral setting. It aligns the crystal three-fold axis (direction 111) along the Y axis, and puts the real space  $c$  axis in the XY plane ( $-X, +Y$  quadrant). Other conventions can be incorporated as well, by inserting the codes in subroutine GTOMDM, which calculates the orthogonalization matrix from

the cell parameters. The deorthogonalization matrix is calculated as the inverse of the orthogonalization matrix.

The AXABZ convention should be used for most calculations. The BYBCX convention was designed for monoclinic space groups in the  $b$ -unique setting to align the crystal  $b$  axis along the Cartesian  $Y$  axis. The AXABZ convention will maintain this alignment and should therefore be used for monoclinic space groups as well.

### III. SPECIFICATION OF LOCAL SYMMETRY

**LOCSymmetry** (RLCAXS(i), i=1, 3), MLCAXS, TLCAXS\*1 [none]

This defines the local symmetry elements in a standard orientation. Only the unique symmetry elements need to be specified for this purpose. For example, for 222 symmetry, only two orthogonal two-folds need to be given. For icosahedral symmetry, only two symmetry axes (one two-fold and one non-orthogonal five-fold, one three-fold and one non-orthogonal five-fold, or two five-folds, etc) need to be given. The rest of the symmetry operators can be automatically generated by the program (see command LOCEXpand). The symmetry operator can be specified as a set of Eulerian angles (TLCAXS='E'), or a set of polar angles (TLCAXS='P') (the interpretation of the angles depends on the convention used, see II), or the Cartesian coordinates of the end point of a vector from the origin (TLCAXS='V'), or as direction cosines (TLCAXS='D'). The default for TLCAXS is Polar. The unique rotation around the axis is specified by MLCAXS, either as an angle or as a divisor of 360. If the axis is specified as polar angles, the information from the polar angle supercedes the value specified for MLCAXS, and a warning will be issued if the information from the two sources do not agree. The maximum number of local symmetry axis that can be specified by this command is given by the MAXLAX parameter.

The program converts the input to a rotation matrix immediately, so the angle convention used to define the local symmetry can be different from that used later in the search.

If no local symmetry elements are defined by the input, the program will carry out ordinary rotation function calculations.

**LOCEXpand** QLCXPD [T]

If QLCXPD is true, the program will generate all the local symmetry operators by pairwise multiplication of all the known operators until no new symmetry element is generated by the process. For this algorithm to work, the positions of the starting local symmetry axes must be specified with sufficient accuracy. If QLCXPD is false, the program will not carry out the above expansion. This is useful in cases where improper rotation axes are being dealt with (for example, a local two-fold with a rotation of  $175^\circ$ ).

A set of polar angles (in the convention specified by POLAR) will be extracted from the rotation matrices and the local symmetry operators are sorted on these angles before they are output. The maximum number of local symmetry operators is given by the MAXLOC parameter.

**LOCInterpolate** QLCINT [F]

This is useful only if the slow rotation function is used to calculate the locked rotation function. If QLCINT is true, an ordinary rotation function will be calculated with the slow rotation function, covering

the entire rotational space (0-360° in  $\theta_1$  and  $\theta_3$  with a grid interval of 2.8125°, 0-180° in  $\theta_2$  with a grid interval of 3.0°). The locked rotation function is then calculated by interpolating among the ordinary rotation function values. (Note that this is the mode of operation if the fast rotation function is used.) If QLCINT is false, the locked rotation function will be calculated directly, with the slow rotation function. For initial searches, QLCINT should be set to true (or the fast rotation function should be used). For fine searches, QLCINT should be set to false (also see command PKFit).

**LOCorient** (ALCORI(i), i=1, 3), TLCORI\*1 [0.0, 0.0, 0.0, E]

The rotation that should be applied to the standard orientation (as defined by LOCSymmetry and LOCUpdate commands) to bring it to that in the crystal is defined with this command. This command is useful only for the locked cross rotation function. The rotation defines the matrix  $[F]$ . The angles can be either Eulerian (TLCORI='E') or polar (TLCORI='P'). The default for TLCORI is Eulerian. The convention of the angles can be specified by POLAR or EULER. The input is immediately converted to a rotation matrix, so the angle convention definition can be temporary.

**LOCUpdate** (ALCUPD(i), i=1, 3), TLCUPD\*1 [0.0, 0.0, 0.0, E]

The orientation of the local symmetry operators as defined by the LOCSymmetry commands can be changed with this command. The updating angles can be either Eulerian (TLCUPD='E') or polar (TLCUPD='P'). The default for TLCUPD is Eulerian. The convention of the angles can be specified by POLAR or EULER. The input is immediately converted to a rotation matrix, so the angle convention definition can be temporary.

#### IV. COMMANDS THAT DEFINE THE FIRST (A) CRYSTAL

The program can deal with the reflection data of three crystals, named A, B and C. It is the A crystal that is rotated in real space to match the B crystal. This means that, in the case of cross rotation search with a model, the calculated structure factors based on the model should be input as the A crystal, and the resulting rotation matrix should be applied directly to the search model. In the case of self rotation search, the A crystal information is copied into the B crystal.

**ACELL-Parameters** (CELL(i, 1), i=1, 6) [1.0, 1.0, 1.0, 90.0, 90.0, 90.0]

This inputs the unit cell parameters of the A crystal. The parameters can be either in real or reciprocal space. The angles can be either in degrees or in cosines.

**ASYMmetry** [none]

This inputs the space group symmetry operators of the A crystal, in the same format as the International Table. Positions related by centering should not be input. Positions related by inversion or mirror planes are not supported by the program. If no symmetry card is given, the program defaults to  $P1$ . The identity operation is assumed and need not be specified.

The space group symmetry can also be specified by giving the symbol of the space group (for example P212121) or the number of the space group (for example 19). For monoclinic space groups, only the  $b$ -unique





This specifies that the input structure factors for the C crystal should be raised to the power of CPOWER to calculate the intensities of the reflections.

**REJECTION**-criteria — CSGCUT, CCUTLO, CCUTHI

This specifies the screening criteria for the C crystal data.

**SYMMETRY** —

This defines the space group symmetry of the C crystal.

The program will read in the reflections and save only the large terms, selected based on the CUTOFF (see command CUTOFF).

## VII. SPECIFICATION OF SEARCH PARAMETERS

**BOX**Size (IBXHKL(i), i=1, 3) [3, 3, 3]

This specifies the interpolation box size around each rotated reflection. The size of the box should be big enough to cover the maximum of the G function. At the same time, it should not be too big because the calculation time is proportional to the volume of the box. For most cases, a  $3 \times 3 \times 3$  box should be large enough. A bigger box may be used in the case of centered lattices due to the systematic absences. The maximum dimension of the box is specified by the MAXBOX parameter.

**CROSS**-search QCROSS [F]

This specifies that the program should perform cross rotation calculations. QSELF (see command SELF-search) and QCROSS are mutually exclusive. Specifying QSELF to be true will disable QCROSS automatically. Due to its default, QCROSS will need to be set to true for cross rotation searches. It also must be true if cross rotation calculations need to be performed by the commands LOCROtate, and MATRix. If QCROSS is false, the program will carry out self-search calculations.

**GEVAL**uation IGEVAL [2]

This specifies how the G function values should be calculated. Currently supported options are – IGEVAL=1, which means the G values will be obtained from a table look-up based on the value of  $r^2$ ; IGEVAL=2, which means the G values will be sampled in the interpolation box at an interval of DHKL in  $h, k,$  and  $l$  (DHKL is a program parameter, currently set to be 0.1); IGEVAL=3, which means the nearest integer of the rotated index will be used in the calculation (fastest way of getting the G value, but with the most error. Recommended only for checking the symmetry of the rotation function); and IGEVAL=4, which means the G values will be calculated based on the formula from the distance ( $r$ ). See the paper by Rossmann & Blow for detailed description of the G function.

**MAP**File MAPFIL\*72 [none]

If a file name is specified by this command, the program will dump the rotation function values as a formatted file. Default is that the map values will not be dumped. This file may be useful for later processing with other programs (see the command PEAK).

**OANGLE** OANGLE\*1, OTYPE\*3 [P, XZK]

The rotation function peaks will also be converted to Eulerian (OANGLE='E') or polar (OANGLE='P') angles for output. OTYPE specifies the convention that should be used with the Eulerian or polar angle (see II). Therefore, two sets of angles are output corresponding to each peak in the rotation function map – the first set is defined by the SANGLE command, the second defined by this OANGLE command.

**PEAK-Cutoff** PKCUT, NLIST, MAPFIL\*72 [3.0, 40]

The program will perform a peak search in the resulting rotation function map. In this peak search, grid points whose values are less than  $PKCUT \times \sigma$  (where  $\sigma$  is the standard deviation of the map values) over the map average will be skipped. NLIST peaks will be printed at the end of the peak search. The maximum number of peaks that can be saved is given by the MAXPKS parameter.

If a file name has been specified for MAPFIL, the program will read the map values from that file and perform a peak search. In this case, no self or cross rotation search will be carried out.

**PKFIT** NPKFIT, PKFCUT [5, 1.5]

The program can automatically carry out fine searches, with the slow rotation function, for the top NPKFIT peaks in the ordinary or locked rotation function. Specifying this command will also set the logical QPKFIT to true, which defaults to false. If a negative number is input for NPKFIT, QPKFIT will be set to false. PKFCUT is the large-term cut-off that will be used by the slow rotation function. The program will first search in  $1^\circ$  intervals to locate the peak. Then the grid interval will be lowered to  $0.5^\circ$ . The peak positions obtained from these fine searches can be used directly by the PIPC and GLTF commands.

**PRERotation** (AUVW(i), i=1, 3), (BUVW(i), i=1, 3) [0]

This command performs a “pre-rotation” which aligns a direction in the A crystal, specified by AUVW, with a direction in the B crystal, specified by BUVW. This command can be used only if a cross-rotation search is being performed. The direction can be specified either as a set of fractional coordinates, in which case the vector from the origin to this point will define the direction, or as a set of polar angles –  $\phi$ ,  $\psi$ , and the third field should be ‘P’olar or a number less than -999.0. The polar angle convention is taken from the POLAR variable and the orthogonalization convention is taken from ORDOR. The subsequent search will be carried out in polar angles. Only  $\kappa$  will be allowed to change,  $\phi$  and  $\psi$  are determined by BUVW.

**RADIus** RADIUS [30.0]

This specifies the radius of integration, in Å.

**RCUToff** RCUTLO, RCUTHI [0, 20]

RCUTLO has been implemented mainly as a way of saving computation time.

In the ordinary rotation search calculated by the slow rotation function, if the rotation function value at a search grid point is less than RCUTLO (default is 25 for self-search and 500 for cross-search), all the neighbors of this grid point is flagged and will be removed from further calculations. These grid points are output in the map file (see command MAPFile) with a value of -999. If there are more than 2500 grid points in the search, the program will determine RCUTLO automatically and carry out a coarse search through the grid points first and remove those grid points that are unlikely to give high rotation function values.







supplied with the AOB`S` command. The program will automatically translate the model to place its center at the origin.

#### **OUtput** – COROUT\*72.

The atomic coordinates after PC refinement can be written out. The default file name is rigid.pdb. The coordinates for all the rotation function peaks that are used in the PC refinement are written to this file. There are comment lines at the beginning of each segment of coordinate entries defining which rotation function peak they correspond to.

**GROUP** IGROUP, RES1\*6, RES2\*6 []

The first step of the PC refinement is always an overall refinement, using the entire molecule as a rigid body. This command can be used to define smaller groups that will be used in the second step of the PC refinement. IGROUP defines the number of the group, and RES1 and RES2 defines the beginning and ending residue number for the group. The command can be repeated to specify different groups or groups with more than one residue ranges. Residues that are not selected by any of the groups will not be changed by the refinement. Chain names are supported and should be separated from the residue name with /, |, or . The question mark ? can be used as a wild card and matches to any character. For example, GROUp 1 A:???? A:????, GROUp 2 B:???? B:???? will define two groups for chains A and B.

**P1PC**-refinement NPCPKS, NPCCYC, PCCUT [5, 10, 1.0]

The program will carry out a Patterson-correlation refinement for the first NPCPKS peaks in the ordinary cross rotation function. NPCCYC specifies the number of cycles for each refinement step. PCCUT specifies a large-term cutoff that should be used with the PC-refinement. Default is that the PC-refinement will not be done.

## X. LOCKED TRANSLATION FUNCTION.

**GLTF**unction OPTION\*4 []

This activates the locked translation function. For this function to work, a locked cross rotation search (either with slow or fast rotation function) must have been carried out first. The peaks in this search can then be used for the locked translation function (see option RFPEak below). The atomic coordinates, corresponding to the A crystal reflection data in the search, should be input with the COORdinate command (see the section on P1PC refinement above).

The program will do a peak search through the translation function and lists the top 10 peaks in the map. The maximum value of the function is scaled to 1000. The peak cut-off for this search is set at  $3\sigma$  or 400, whichever is smaller. If packing check is not used during the calculation (see option PACK below), the position with a height of 1000 may not be present in the peak listing as it may have packing problems. The program will output the atomic model for the entire assembly corresponding to the top peak in the locked translation function to a file. A name can be specified for this file with the COORdinate OUtput command (see above). If more than one rotation peaks are used, the file will contain separate assemblies, each identified with the rotation function peak number. This model for the assembly should then be input

to an ordinary translation function program (for example, TF) to find the center of the assembly in the unit cell.

OPTION is a four-letter keyword that define the function of the command. Currently supported options are (in alphabetical order) –

**CUTOFF** – GLTFCT.

The large-term cutoff that should be used for the locked translation function. Default is 1.5.

**DIRECT**-summation – QDRECT.

This specifies whether the locked translation function should be evaluated by direct summation. Default for QDRECT is false, meaning the function will be evaluated by FFT. If the local symmetry is high (point group 432 or higher), the program will force direct summation.

**MAP**File – LTFMAP\*72.

This specifies a name for a file where the locked translation function map will be dumped. Not implemented yet.

**OUTP**ut – NTFOUT, NTFOUP.

This specifies whether the program will output all the atoms (NTFOUT=1) or only the Ca atoms (NTFOUT=2) for the locked TF solutions. The default for NTFOUT is 1. NTFOUP specifies the number of solutions that should be written out for each orientation. Default for NTFOUP is one, meaning only the top solution will be written out.

**OVER**lap – NOVLP, DCUTTF.

This specifies the maximum number of close Ca - Ca (or P - P for nucleic acids) contacts that are allowed in the packing check. The distance cut-off for this check is given by DCUTTF. The default for NOVLP is 40. The default for DCUTTF is 3Å, which should be appropriate for protein structures. For nucleic acid structures, try using DCUTTF of 5Å. The program will try to save only 200 Ca atoms, which may lead to the selection of every second, or third Ca atoms only. In such cases the value of NOVLP will be changed automatically by the program.

**PACK**ing-check – QPACK.

This specifies whether packing of the monomers in the assembly should be checked during the calculation. Default for QPACK is false. This is coupled with the default for QDRECT of false. If direct summation is used, QPACK should be set to true. If this packing check is not done during the calculation, it will be performed during the peak search. The packing check during the peak search also examines the steric contacts among the assemblies that are related by unit cell translations. If the number of contacts for either packing check is greater than a pre-set number (see option OVERlap), the grid point will be ignored. The number of contacts found within the assembly is listed under 'Overlap'. The number of contacts found among the assemblies is given under 'Cell' in the peak listing.

**RADI**us – RADMAX, RADMIN.

This defines the maximum and the minimum lengths of the translation vector. Grid points in the search region (see option SLIMts below) that are more than RADMAX angstrom away from the origin will be ignored. The default for RADMAX is 30Å. Grid points in the search region that are less than RADMIN angstrom away from the origin will be ignored. The default for RADMIN is 5Å.

**RFPE**ak – NTFPKS.

The top NTFPKS peaks in the locked cross rotation function will be used for the locked translation function. The program will do a separate calculation for each rotation peak. Alternatively, if NTFPKS has been specified as a number between -1 and 1, the program will select all the locked cross RF grid points with heights greater than NTFPKS times the maximum value of the RF for the locked TF calculation. This is similar to the combined molecular replacement option in the TF program of this package.

**SLIMits** – I, BEGIN, END, INCREMENT.

This defines the search region for the translation vector. The integer I can take the value 1, 2, or 3, corresponding to the Cartesian coordinates X, Y, or Z. The values should be given in angstrom units.

## XI. MISCELLANEOUS COMMANDS

**ANGLE** X, Y, Z, TANGLE\*1 []

This defines an angle to the program. Currently this information is used only by the XCORrelate command. See command LOCRotate for more information.

**LOCAsu** (RLCASU(i), i=1, 3) [10.0, 20.0, 30.0]

This command instructs the program to explore the symmetry of the locked self rotation function (QSELF should be true). The program first updates the local symmetry matrices based on the angles specified by RLCASU (similar to LOCUpdate), it then carries out a grid search, using parameters defined by the SLIMits commands and the SANGLE command, to find other sets of angles that are related to RLCASU by the symmetry. Internally, the command LOCAsu assigns a true value to the logical QLCASU, which defaults to false. Therefore, LOCAsu command must be supplied, even though you want the default angles for RLCASU (i.e., you can input a line that says “LOCA” only).

Symmetry in the locked cross rotation function can be determined analytically and is not supported as yet.

**LOCFit** ANGINC, ZANGLE [0.5, 2.5]

This command performs an ordinary self rotation search for each of the local symmetry operators at a given orientation. The orientation can be derived by updating the standard orientation with the LOCUpdate command. The program searches ZANGLE degrees to either side of the known  $\phi$  and  $\psi$  positions, with ANGINC as the interval between grid points. Internally, the commands sets the logical variable QLCFIT to true, which defaults to false.

**LOCRotate** (ALCSHO(i), i=1, 3), TLCSHO\*1 [0]

If QSELF is true, this command takes a set of angles and updates the local symmetry operation matrices based on the rotation matrix of the input angles. The angles can be either Eulerian (TLCSHO=‘E’) or polar (TLCSHO=‘P’). The default for TLCSHO is Eulerian. The convention of the angles can be specified by using the EULER or the POLAR commands (see II). If reflection data for crystal A have also been supplied, the

program will obtain the rotation function value at each of the updated positions. The program also produces a stereographic representation of the set of updated local symmetry rotation axis. A set of polar angles are extracted from each of the updated matrices using the convention specified by the POLAR command. The top half of the sphere ( $z > 0$  in the case of XYK polar angles, so  $\pi < \phi < 2\pi$ , and  $y > 0$  in the case of XZK polar angles) is projected. The PostScript file is named STEREO.PS.

If QCROSS is true, the updated matrix is calculated as  $[F][I_i][E]$ , where  $[E]$  is the matrix based on the input angles. A set of angles (polar if SANGLE='P', Eulerian if SANGLE='E') is extracted from the matrix. If reflection data has also been input for the A and the B crystals, the program will calculate the rotation function value corresponding to each of the updated matrices.

The maximum number of angles that can be input by this command is specified by the parameter MAXSHO.

**MATRIX** (AMTSHO(i), i=1, 3), TMTSHO\*1 [0]

This commands takes a set of angles and outputs the corresponding rotation matrix. The variables AMTSHO and TMTSHO are similar to those for command LOCRotate. The program calculates the rotation matrix immediately after this command is specified. Therefore, a temporary angle convention definition can be specified. If a pre-rotation has been applied (through the PRERotate command), the program will concatenate the two rotation matrices and extract a set of angles based on the new matrix. The angle type is specified by SANGLE, and the angle convention by EULER or POLAR. The program then applies the orthogonalization and deorthogonalization matrices. Therefore, the correct cell dimensions should be input to the program. If reflection data has been input, the program will output the contribution of the top 50 large terms to the final rotation function value. In this case, IGEVAL (see command GEVAluate) is set to 2. The maximum number of angles that can be input by this command is specified by the parameter MAXSHO.

**SYMShow** (ASYMSH(i), i=1, 3), TSYMSH\*1 [0]

This commands takes a set of angles and outputs all the angles that are related to the input set by the symmetry of the ordinary rotation function. The variables ASYMSH and TSYMSH are similar to those for command LOCRotate. The program calculates the rotation matrix immediately after this command is specified. Therefore, a temporary angle convention definition can be specified. Calculations with both self and cross rotation functions are possible. The program will output the angles as Eulerian if SANGLE is 'E' or as polar if SANGLE is 'P'. The angle convention is given by EULER or POLAR. Note that the calculations in this command ignore the local symmetry information. Use the command LOCAsu to explore the symmetry of the locked rotation function. The maximum number of angles that can be input by this command is specified by the parameter MAXSHO.

## XII. ELECTRON DENSITY ROTATION TRANSLATION FUNCTION

**RSRF** OPTION\*1, X, Y, Z, CODE\*1 [none]

This command specifies parameters for the real space (electron density) rotation function (see Appendix D for an introduction). This can be used to maximize the overlap of the electron density of two molecules

related by local symmetry. When `OPTION='R'`, X, Y, and Z will be interpreted as a set of rotation angles relating the molecule in the A crystal to that in the B crystal. The rotation angle type (Eulerian or polar) will be taken from `SANGLE` (command `SANGLE`) and the angle convention will be taken from `EULER` or `POLAR`. When `OPTION='A'`, X, Y, and Z will be interpreted as the center of the molecule in the A crystal, in fractional coordinates. When `OPTION='B'`, X, Y, and Z will be taken as the center of the molecule in the B crystal. Therefore, three separate `RSRF` commands will be needed to define the relationship between the molecule in the A crystal and that in the B crystal. Set the `CODE` to 'S' if a search is to be carried out for the corresponding `OPTION`. Note that only one `OPTION` can have the code 'S' at one time.

The two molecules can reside in the same crystal (self-rotation calculation) or different crystals (cross-rotation calculation). The program will read in phase angles for each reflection. The search limits will need to be defined with the `SLIMs` commands. Note that in this case the limits defined are taken as the offsets from those values defined by the `RSRF` commands. The G function evaluation option (`GEVALuate` option) will be set to 2.

**TFUNction** [0]  
ITFUNC, (TFANGL(i), i=1, 6)

This command calls the translation function routines. The program has an internal logical variable called `QTFUNC`, which defaults to false. Specification of this command sets `QTFUNC` to true. Currently, three types of translation function are supported.

The first, `ITFUNC=1`, searches for the translation element along the local symmetry axis, whose direction is given by the angles `TFANGL(1)`, `TFANGL(2)`, `TFANGL(3)`. (The angle type is specified by the `SANGLE` variable and the angle convention is given by the `POLAR` or `EULER` variable). This function only works for local two-fold axes (M. G. Rossmann, D. M. Blow, M. M. Harding, E. Collier, *Acta Cryst.* **17**, 338, (1964)).

The second, `ITFUNC=2`, searches for the location of the specified local symmetry axis in the unit cell (see Appendix E). In this case, a set of phase angles must be present in the reflection file. Similar to the case for `ITFUNC=1`, `TFANGL(1)`, `TFANGL(2)`, `TFANGL(3)` defines the orientation of the local symmetry axis as a set of polar or Eulerian angles. `TFANGL(4)` defines the translational element (in Å) along this local symmetry axis. The program will break it down to three translation elements along the unit cell edges. Be sure to confirm that the signs of those elements are correct. If not, then the sign of `TFANGL(4)` may need to be reversed.

The third, `ITFUNC=3`, searches for the position that is related to an input point by the local symmetry axis (see Appendix F). This also requires a set of phases to be input to the program. `TFANGL(1)`, `TFANGL(2)`, `TFANGL(3)` defines the orientation of the local symmetry axis. `TFANGL(4)`, `TFANGL(5)`, `TFANGL(6)` defines the input position, in fractional deorthogonalized units.

In all three cases, the program produces a formatted file called `TFUNi.DAT`, where `i=1, 2, or 3`, which contains  $h, k, l, F$ , and  $\phi$  values for a set of reflections. The maximum coefficient ( $F$ ) has been scaled to a value of 1400. These reflections can then be input to any Fourier summation program to obtain the translation function map (in space group  $P1$ ). The maximum number of reflections that can be saved by the program is given by the parameter `MAXREF`.

The program will also try to carry out the Fourier transform directly. A warning will be issued if array limits are exceeded and the program can not perform the transform. A peak search is done through the transform map. The map can also be contoured (command `CNTFile`). In this case, the desired sectioning of the contour should be specified (command `SECTION`). The contour level (command `CNTLevel`) can be

set at 500, 1000, 50, as the program will scale the maximum of the transform to 1000. The contour sections (command CNTSection) can be left as default, which will lead to the contouring of 5 sections containing the top peaks. The map can be written to a file (command MAPFile), which can be used for re-contouring the map later (command PEAK).

### XIII. CONTOUR PLOTS

**CNTFile** **CNTFIL\*72** [none]

If a file name has been specified by this command, the program will produce contour plots for the rotation function map. Default is that the contour plot will not be created.

If an one-dimensional search has been carried out (*i.e.*, only one angle was allowed to vary), the rotation function values will be plotted as a function of the varying angle. In this case, the CNTLevel and CNTSection commands are ignored by the program.

If polar angles were used in the search, the contour plot will be done stereographically in sections of  $\kappa$  (with projection down Z if POLAR is XYK and down Y if POLAR is XZK). To make this projection consistent with the convention used in other plots (see command LOCRotate), the  $\psi$  angle needs to be kept between 0 and 180°; the  $\phi$  angle should lie between 0 and 180° if convention XZK is used, and between 180 and 360° if convention XYK is used (see command SLIMits). Normally, stereographic projection will plot out the bottom half ( $y < 0$ ) of the sphere for XZK polar angles and the top half of the sphere ( $z > 0$ ) for XYK polar angles. If a self rotation function is calculated in polar angles, the three coordinate axes in the stereographic contour plot will be labelled with the appropriate unit cell axes ( $a$ ,  $a^*$ , or  $a//$ , which means the direction is roughly along the unit cell  $a$  axis).

If Eulerian angles were used in the search, the sectioning of the contour plot will be taken from the SECSHN parameter (see command SECTION).

The contouring algorithm assumes continuity in the rotation function map values. This may not be true if a high RCUTLO value (see command RCUToff) has been selected during the calculation. In these cases, there might be some strange contour lines in the plot. To alleviate this problem, the RCUTLO value will need to be reduced.

The maximum number of points in each contour trace is given by the parameter MAXCVS. If the program is stopped because this limit is exceeded, either recompile the program with a larger MAXCVS or raise the starting contour level.

**CNTLevel** (CNTLVL(i), i=1, 3), LINTYP [none]

This command specifies the beginning, ending, and increment for the contour level. LINTYP specifies the line type to be used for the contours in this level. This command can be repeated to obtain different contour levels. The contour levels can be specified as absolute values (with the maximum of the map scaled to 999) or in terms of the  $\sigma$  value of the map. In the latter case, the beginning and ending contour levels must be specified as less than 10 and the increment must be less than 2. For example, specifying 'CNTLevel 1 5 0.5' means the contouring should begin at  $1\sigma$  above the map average and end at  $5\sigma$  above the map average, with increment of  $0.5\sigma$ . The maximum number of contour levels that can be specified is given by the MAXLVL parameter.

Currently 7 different line types are supported. LINTYP=1 is a solid line 0.1 point wide (there are 72 points to an inch). LINTYP=2 is a solid line 0.5 point wide. LINTYP=3 is a solid line 1 point wide. LINTYP=4 is a dashed line (2 on 2 off) 0.1 point wide. LINTYP=5 is a dashed line 0.5 point wide. LINTYP=6 is a dashed line (4 on 1 off) 0.1 point wide. LINTYP=7 is a dashed line 0.5 point wide.

Since it is difficult to predict the optimal contour levels beforehand, it is suggested that the rotation function values be written out with the MAPFile command. Then contour level(s) can be chosen and the rotation function map values can be read in with the command PEAK.

**CNTS**Section ICNTSB, ICNTSE [0, 0]

This specifies the section range for which the contour plot should be created. The numbering of the sections starts with the first section being number 1. The default is that all the sections will be plotted if there are 5 or less sections in the map. If there are more than 5 sections, the program will plot a maximum of 5 sections containing the top peaks in the map. If no peaks are found in the map, the first 5 sections of the map will be plotted.

#### XIV. STORAGE LIMITS

The storage limits for different parameters are defined in a PARAMETER statement in the Fortran INCLUDE file GLRF.CMN. The program will need to be recompiled if any of these limits is exceeded. Especially, the parameters MAXH, MAXK, and MAXL should be adjusted to best match the cell parameters under study. The following table lists a set of reasonable values for the parameters. The actual values used are reported at the end of the output file.

Parameter	Limit
MAXBIG	80000
MAXBOX	5
MAXGRD	1000000
MAXH	75
MAXK	75
MAXL	75
MAXLAX	10
MAXLOC	100
MAXPKS	1000
MAXREF	100000
MAXSHL	10
MAXSHO	10
MAXATM	12000
MAXLVL	5
MAXLIN	7

## Appendix A.

### New Features

#### New Features of Version 3.4 — June, 1996

- . The locked TF can now use grid points in the locked cross RF that are above a specified threshold. Also, an option to output only the Ca atoms of the locked TF solutions is implemented (GLTF OUTPut).
- . The fast RF can now be used for any Euler angle conventions or even polar angles. The program will do interpolation if necessary.
- . Bug fix in ORFLRF for the interpolation.

#### New Features of Version 3.3 — April, 1996

- . Calculation of fast rotation functions with normalized structure factors is supported. New command NORMalize.
- . Calculation of the self rotation function in polar angles with the fast RF, by interpolation.
- . Correlation of ordinary cross RF results with the self RF. New command XCORrelate.
- . Bug fixes and documentation updates.

#### New Features of Version 3.2 — September, 1995

- . A general locked translation function is implemented. A new command GLTF is introduced.
- . The program can automatically carry out fine searches, with the slow rotation function, for the top peaks of the RF. A new command PKFIt is implemented. These fine search results are used by P1PC and GLTF commands.
- . An option of calculating the locked RF by interpolating among ordinary slow RF values is implemented. A new command LOCInterpolate is introduced.
- . The program will select both Ca and P atoms for packing check in the locked translation function.
- . Modified the coordinate I/O routines to fully support the PDB format.
- . Implemented a new orthogonalization convention, CXCAZ.
- . Various bug fixes.
- . Updated the documentation.

#### New Features of Version 3.1 — June, 1995

- . The Patterson correlation refinement of ordinary cross RF peaks is now possible. The correlation that is maximized is different from that used in X-Plor. New commands COORdinate, GROUp, and P1PC.
- . The Crowther fast rotation function is now fully implemented. It can be used to calculate locked as well as ordinary self or cross rotation functions.

- . The unique rotational space for Eulerian searches can be assigned automatically for ordinary self and cross RF (a special feature of the command SLIMits). The program can also try to assign the limits for the locked RF.
- . A new algorithm for the calculation of the slow RF is implemented, which will determine RCUTLO (command RCUToff) automatically and can lead to a three-fold speed-up. The implementation of RCUTHI (command RCUToff) has been changed.
- . A new definition of the locked cross RF is used, which should simplify the definition of the unique rotational space.
- . FFT routines have been incorporated to perform the transform from calculations done with the command TFUNction directly.
- . The angles in the peak listing in the print file are now labeled. The three coordinate axes in the stereographic projection are labeled with the appropriate unit cell axis.
- . Bug fixes.
- . Updated the documentation.

## Appendix B.

### Example Inputs

Example input files to this program are provided together with the program distribution. This appendix shows examples for blocks of related inputs. The required portion of each command is given in upper case.

#### General Set-up

TITLe RF calculation  
 COMMeNt anything goes here  
 PRINt glrf.prt  
 EULER zxz  
 POLAR xzk  
 ORTHogonalization axabz

#### Define Crystal A

ACEl 90 90 90 90 90 90  
 ASYMeTtry p212121  
 AOBS-file fobs.dat  
 AFORmat 3i4, 2f8.2  
 ACUTOff 1 1 0  
 APOWEr 2  
 NSHELL 8  
 ORIGIn true

#### Slow Self Rotation Function

SELF true  
 CROsS false  
 FAST false  
 CUTOff 1.5  
 BOXSizE 3 3 3  
 GEVALuation 2  
 RADIus 20  
 RESOLution 10 3.5  
 SANGLe polar  
 OANGLe euler zxz  
 SLIMits 1 0 180 3  
 SLIMits 2 0 180 3  
 SLIMits 3 180 180 2  
 MAPFile srf.map

#### Fast Cross Rotation Function

SELF false  
 CROsS true  
 FAST true  
 RADIus 30  
 RESOLution 10 3.5  
 CUTOff 0.2  
 SANGLe euler  
 OANGLe euler zxz  
 SLIMits 270 270 0  
 MAPFile xrf.map

#### Peak Search and Fitting

PEAK 3 50  
 PKFIt 10 1.5

#### Contour Plots

CNTFile glrf.ps  
 CNTLevel 500 1000 50  
 SECTion 123

#### Non-crystallographic Symmetry

LOCSymmetry 1 0 0 2 vector  
 LOCSymmetry 1 1 1 3 vector  
 LOCEXpand true

#### Real Space Rotation Function

RSRF rotation 20 30 40 search  
 RSRF a-center 0.1 0.2 0.3  
 RSRF b-center 0.4 0.6 0.5

**Read in Old Map File**

PEAK 3 30 glrf.map  
CNTFile glrf.ps  
CNTLevel 500 1000 50  
SECTion 123

**Fast Locked Cross RF**

LOCSymmetry 1 0 0 2 vector  
LOCSymmetry 0 1 0 2 vector  
LOCExpand true  
LOCorient 10 20 30 euler  
SELF false  
CROSSs true  
FAST true  
RADIus 30  
RESOLution 10 4  
CUTOFF 0.2  
SANGLe euler  
OANGLe euler zyz  
SLIMits 1 0 180 3  
SLIMits 2 0 90 3  
SLIMits 3 0 360 3

**Patterson Correlation Refinement**

COORdinate input model.pdb  
COORdinate boverall 30  
COORdinate output rigid.pdb  
GROUp 1 1 60  
GROUp 1 181 250  
GROUp 2 61 180  
P1PC 10 5 1.5

**Locked Translation Function**

COORdinate input model.pdb  
COORdinate boverall 30  
COORdinate output gltf.pdb  
GLTF cutoff 1.5  
GLTF direct false  
GLTF overlap 10 3  
GLTF packing false  
GLTF radius 30  
GLTF rfpeak 10  
GLTF SLIMits 1 -30 30 1  
GLTF SLIMits 2 -30 30 1  
GLTF SLIMits 3 -30 30 1

## Appendix C.

### The Cross-Locked Rotation Function

In many cases a protein sample has been crystallized into two (or more) different space groups. The orientational relationship between a molecule in the first crystal form and one in the second can be determined by a cross-rotation function between the two crystal forms. Thus, we have the following relationship,

$$y_C = [\alpha_C][R_{BC}][\beta_B]y_B,$$

where the two crystal forms are called B and C.

Now a cross rotation function can be calculated between a search model and crystal forms B and C simultaneously. The two individual rotation functions are,

$$y_B = [\alpha_B][R_{AB}][\beta_A]y_A$$

$$y_C = [\alpha_C][R_{AC}][\beta_A]y_A$$

Noting the relationship between the B and the C crystals, we should have

$$\begin{aligned} y_C &= [\alpha_C][R_{BC}][\beta_B]y_B \\ &= [\alpha_C][R_{BC}][\beta_B][\alpha_B][R_{AB}][\beta_A]y_A \\ &= [\alpha_C][R_{BC}][R_{AB}][\beta_A]y_A \end{aligned}$$

Therefore,

$$[R_{AC}] = [R_{BC}][R_{AB}]$$

The cross-locked rotation function (or triple rotation function) can be defined as the product of the rotation function values corresponding to matrix  $[R_{AB}]$  between A and B crystal and matrix  $[R_{BC}][R_{AB}]$  between A and C crystal.

The above derivation can be easily generalized to cases where there are more than one rotational relationships between crystals B and C, and to cases where there are more than two crystal forms (quadruple rotation function ...).

## Appendix D.

### The Real Space Rotation Function

This function is defined as the product of the electron density for two copies of the same molecule. It is dependent on both the rotational and the translational relationship between the two copies (and in this sense the function is more than just a rotation function).

The relationship between the two copies of the molecule can be written as,

$$x_B = [C]x_A + d.$$

If  $S_A$  and  $S_B$  are the centers of the molecule in crystals A and B, respectively, then

$$S_B = [C]S_A + d.$$

The correlation between the two molecular copies should be a function of  $[C]$ ,  $S_A$  and  $S_B$ ,

$$\begin{aligned}
R([C], S_A, S_B) &= \int_{(|x-S_A| \leq R)} \rho(y)\rho(x) dx \\
&= \int_{\Omega} \sum_p F_p e^{-2\pi i p y} \sum_h F_h e^{-2\pi i h x} dx \\
&= \sum_p \sum_h F_p F_h \int_{\Omega} e^{-2\pi i p([C]x+d)} e^{-2\pi i h x} dx \\
&= \sum_p \sum_h F_p F_h e^{-2\pi i p d} \int_{\Omega} e^{-2\pi i (h+p[C])x} dx \\
&= \sum_p \sum_h F_p F_h e^{-2\pi i p d} G_{hp} e^{-2\pi i (h+p[C])S_A} \\
&= \sum_p F_p e^{-2\pi i p([C]S_A+d)} \sum_h F_h G_{hp} e^{-2\pi i h S_A} \\
&= \sum_p F_p e^{-2\pi i p S_B} \sum_h F_h G_{hp} e^{-2\pi i h S_A} \\
&= \sum_p F_p e^{-2\pi i p S_B} A_p e^{i B_p} \\
&= \sum_{\substack{p \\ (l \geq 0)}} \left( F_p A_p e^{i \phi_p} e^{-2\pi i p S_B} e^{i B_p} + F_p A_p e^{-i \phi_p} e^{2\pi i p S_B} e^{-i B_p} \right) \\
&= 2 \sum_{\substack{p \\ (l \geq 0)}} F_p A_p \cos(\phi_p + B_p - 2\pi p S_B)
\end{aligned}$$

## Appendix E.

### The Location of a Local Symmetry Axis

Given the orientation of a symmetry axis and the translational elements along the axis, the location of the symmetry axis in the unit cell can be determined by correlating the electron density of the two molecules that are related by the symmetry axis.

The local symmetry axis can be written as

$$x' = [C]x + d_{\parallel} + d_{\perp},$$

where  $d_{\parallel}$  is the translation element along the direction of the symmetry axis and  $d_{\perp}$  the translation element in the plane perpendicular to the symmetry axis. If  $s$  is a point lying on the symmetry axis, it follows that

$$s' = s + d_{\parallel},$$

and

$$s = [C]s + d_{\perp}.$$

The translation function is defined as a function of  $s$ ,

$$\begin{aligned} T(s) &= \int_{(|x-s| \leq R)}_{\Omega} \rho(x)\rho(x') dx \\ &= \int_{\Omega} \sum_h F_h e^{-2\pi i h x} \sum_p F_p e^{-2\pi i p x'} dx \\ &= \sum_h \sum_p F_h F_p \int_{\Omega} e^{-2\pi i h x} e^{-2\pi i p ([C]x + d_{\parallel} + d_{\perp})} dx \\ &= \sum_h \sum_p F_h F_p e^{-2\pi i p (d_{\parallel} + d_{\perp})} \int_{\Omega} e^{-2\pi i (h + p[C])x} dx \\ &= \sum_h \sum_p F_h F_p e^{-2\pi i p (d_{\parallel} + d_{\perp})} G_{hp} e^{-2\pi i (h + p[C])s} \\ &= \sum_h \sum_p F_h F_p G_{hp} e^{-2\pi i p d_{\parallel}} e^{-2\pi i p ([C]s + d_{\perp})} e^{-2\pi i h s} \\ &= \sum_h \sum_p F_h F_p G_{hp} e^{-2\pi i p d_{\parallel}} e^{-2\pi i (h + p)s} \end{aligned}$$

## Appendix F.

### Positions Related by Local Symmetry

Given the orientation of a symmetry axis and an input position belonging to one of the molecules related by the symmetry axis, other positions related to the input position by the local symmetry axis can be calculated.

The local symmetry axis can be written as

$$x' = [C]x + d.$$

If  $s$  is the input position, a translation function can be defined in terms of its symmetry-related position  $s' = [C]s + d$ ,

$$\begin{aligned}
 T(s') &= \int_{\substack{\Omega \\ (|x-s| \leq R)}} \rho(x)\rho(x')dx \\
 &= \int_{\Omega} \sum_h F_h e^{-2\pi i h x} \sum_p F_p e^{-2\pi i p x'} dx \\
 &= \sum_h \sum_p F_h F_p \int_{\Omega} e^{-2\pi i h x} e^{-2\pi i p ([C]x+d)} dx \\
 &= \sum_h \sum_p F_h F_p e^{-2\pi i p d} \int_{\Omega} e^{-2\pi i (h+p[C])x} dx \\
 &= \sum_h \sum_p F_h F_p e^{-2\pi i p d} G_{hp} e^{-2\pi i (h+p[C])s} \\
 &= \sum_h \sum_p F_h F_p G_{hp} e^{-2\pi i h s} e^{-2\pi i p ([C]s+d)} \\
 &= \sum_h \sum_p F_h F_p G_{hp} e^{-2\pi i h s} e^{-2\pi i p s'}
 \end{aligned}$$



## Appendix H.

### Handling of Logicals in Subroutine CHEKIN

	QSELF	QCROSS	Reflection Data
Self Search	T	F	A
Cross Search	F	T	AB
LOCAsu	T	F	none
	F	T	none
LOCFit	T	F	A
LOCRotate	T	F	none, or A
	F	T	none, or AB
MATRix	T	F	none, or A
	F	T	none, or AB
PRERot	F	T	none, or AB
TFUNction	T	F	A